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THERMAL MODEL OF A SOLAR COOKER JORHEJPATARANSKUA

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Abstract

In this paper we obtain a mathematical thermal model to explain the behavior of the solar cooker: "Jorhejpataranskua". The model is shown in terms of a coupled system of three non-linear differential equations. We calculate temperatures for the fluid, the reflectors and the container of the solar cooker, and then compare the numerical results obtained with the model to measurements in field-testing operations and obtained results that agree with the experimental data. Finally, we use numerical results to calculate the cooking power and standardized cooking power of the solar cooker for two different containers.

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Keywords: Solar cooker, thermal model; mathematical model

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1. Introduction

The development of technologies for solar thermal systems is sometimes difficult to carry out because of adverse environmental conditions, particularly for solar cookers, where it instrumentation and equipment are necessary for testing protocols. Therefore, we propose a mathematical thermal model based on the study of thermal physics of a solar cooker, "Jorhejpatarnskua" [1,2], which consists of a Compound Parabolic Concentrator revolution (CPC) with a cylindrical aluminum container as the absorber. In order to estimate the water temperature experimentally, we built an experimental design with an arrangement of lamps to simulate solar radiation, similar to the one considered in [3]. Other research on solar cookers can be found in [4], where the authors show the performance and temperature distribution of a box-type solar cooker. They also consider a mathematical model of a transitional type and the corresponding experimental work. In [5], the authors develop a parametric model for a solar cooker; this model predicts cooking power through controlled parameters and uncontrolled variables. The researchers in [6] obtained a mathematical model for a box-type solar cooker, which helps to establish the elements of the parameters and their influence on the heating process for their solar cooker. In [7], the authors studied the effects of using a flat reflector on a box-type solar cooker. Their results establish advances in increasing the heating temperature and efficiency. Moreover, there is similar research for box-type and concentration-type solar cookers (see, for example[8], [9], [10] and [11]).

The paper is organized as follows:

- Description of the thermal model
- Analysis of the heat transfer equations
- Experimental design
- Results and comparison with field tests

Nomenclature

$Q_{1\mathrm{rad}}$ Incident radiant heat flux over the absorber container (W)
$Q_{2\mathrm{rad}}$ Radiant heat flux between container and sky (W)
$Q_{3\mathrm{conv}}$ Convective heat flux between container and environment (W)
$Q_{3\mathrm{conv2}}$ Convective heat flux between container and air inside cooker (W)
$Q_{4\mathrm{rad}}$ Radiant heat flux between container and reflectors (W)
$Q_{5\mathrm{rad}}$ Incident radiant heat flux over reflectors (W)
$Q_{6\mathrm{conv}}$ Convective heat flux between reflectors and environment (W)
$Q_{7\mathrm{rad}}$ Radiant heat flux between reflectors and sky (W)
$Q_{8\mathrm{conv}}$ Convective heat flux between container's surface and its inside (W)
$Q_{9\mathrm{rad}}$ Radiant heat flux from container to fluid (W)
m_r Mass of container (kg)
m_{rf} Mass of reflector sheet (kg)
m_f Mass of water (kg)
c_r Specific heat of the aluminum container ($\frac{J}{Kkg}$)

c_{rf} Specific heat of the aluminum reflectors ($\frac{J}{Kkg}$)

c_f Specific heat of water ($\frac{J}{Kkg}$)

T_r Mean temperature of the container (K)

T_{rf} Mean surface temperature of reflector sheet (K)

T_f Mean water temperature (K)

T_{amb} Ambient temperature (K)

T_{sky} Temperature of the sky (K)

t Time (sec)

A_{rf} Area of solar collector (m^2)

A_r Area of the container (m^2)

α Absorbance of the absorber

α_{rf} Absorbance of the reflectors

η_0 Thermal efficiency

ρ_m Reflectance of reflector sheet

n Mean number of reflections into the CPC

ε_r Emittance from the absorber

ε_{rf} Emittance from the reflectors

$h_{r,amb}$ Convective heat coefficient between absorber and environment ($\frac{W}{Km^2}$)

$h_{r,int2}$ Convective heat coefficient between absorber and air inside the solar cooker ($\frac{W}{Km^2}$)

$h_{rf,amb}$ Convective heat coefficient between reflectors and environment ($\frac{W}{Km^2}$)

$h_{r,inte}$ Convective heat coefficient between container and fluid ($\frac{W}{Km^2}$)

I_D Direct irradiance ($\frac{W}{m^2}$)

I_R Reflected irradiance ($\frac{W}{m^2}$)

σ	Stefan Boltzmann constant ($\frac{W}{m^2K^4}$)
P_r	Cooking power (W)
$P_{s\tan dard}$	Standardized cooking power (W)
ΔT	Temperature difference between fluid and environment (K)

2. **Description of the thermal model**

We can describe the mathematical thermal model in terms of a coupled system of non-linear differential equations for the mean temperature of the absorber container, reflective sheets and fluid. We obtain the equations by using energy conservation to the total heat flux over the solar cooker.

2.1. Energy balance equations

2.1.1. Heat transfer equation of the container

We apply energy conservation to heat transfer inside the absorber container to obtain the differential equation for mean absorber container temperature:

$$m_r c_r \frac{dT_r}{dt} = Q_{1\mathrm{rad}} - Q_{2\mathrm{rad}} - Q_{3\mathrm{conv}} - Q_{4\mathrm{rad}} - Q_{8\mathrm{conv}} - Q_{9\mathrm{rad}} - Q_{3\mathrm{conv2}} \quad (1)$$

Where

$Q_{1\mathrm{rad}} = A_r \alpha I_D + A_{rf} I_R$	$Q_{2\mathrm{rad}} = A_r \varepsilon_r \sigma \left(T_r^4 \quad T_{cielo}^4\right)$
Stefan Boltzmannconstant $\sigma = 5.669 X 10^{-8} \frac{W}{m^2C^4}$	Emittance of absorber $\varepsilon_r = 0.5$
$Q_{2\mathrm{rad}} = A_r \varepsilon_r \sigma \left(T_r^4 \quad T_{cielo}^4\right)$	$T_{sky} = 0.0552 T_{amb}^{1.5}$
$Q_{3\mathrm{conv}} = A_r h_{r,amb} \left(T_r \quad T_{amb}\right)$	$Q_{3\mathrm{conv2}} = A_r h_{r,int2} \left(T_r \quad T_{int2}\right)$
$T_{int2} = \frac{T_r + T_{rf}}{2}$ $T_{inte} = \frac{T_r + T_f}{2}$	$Q_{4\mathrm{rad}} = A_r \varepsilon_r \sigma \left(T_r^4 \quad T_{rf}^4\right)$
$Q_{8\mathrm{conv}} = A_r h_{r,inte} \left(T_{inte} \quad T_r\right)$	$Q_{9\mathrm{rad}} = A_r \varepsilon_r \sigma \left(T_r^4 \quad T_f^4\right)$

2.1.2. Heat transfer equation of reflectors

This case is similar to the previous one; we only applied energy conservation of the reflector sheets to obtain:

$$m_{rf} c_{rf} \frac{dT_{rf}}{dt} = Q_{5\mathrm{rad}} + Q_{4rad} - Q_{6\mathrm{conv}} - Q_{7\mathrm{rad}} + Q_{3\mathrm{conv}2} \quad (2)$$

Where

$$Q_{7\mathrm{rad}} = A_{rf} \varepsilon_{rf} \sigma \left(T_{rf}^4 - T_{sky}^4\right)$$

2.1.3. Heat transfer equation of fluid

Finally, we have:

$$m_f c_f \frac{dT_f}{dt} = Q_{8\mathrm{conv}} + Q_{9\mathrm{rad}} \quad (3)$$

Equations (1), (2) and (3) represent a coupled system of non-linear differential equations for the variables T_f , T_{rf} and T_r . We take the values of the numerical constants from [12].

3. Solution of the coupled system of nonlinear differential equations

In order to solve equations (1), (2) and (3) numerically, we use Mathematica 8, a scientific software that is capable of generating numerical solutions for general systems of non-linear differential equations of arbitrary order with high accuracy. As it is necessary to express the equations in physical units, for this case we use Watts (W) for the heating , degrees (℃) for temperature, and seconds (s) for time.

Equations (1), (2) and (3) are first order differential equations; therefore, we need to give initial conditions for the unknown variables:

$T_r(t=0) = T_{amb}$, $T_{rf}(t=0) = T_{amb}$, $T_{rf}(t=0) = T_{amb}$,

where T_{amb} is the ambient temperature.

Finally, we use Mathematica's command NDSolve to obtain the numerical results for the variables , and (see Appendix A).

3.1 Cooking power estimation

We use the numerical value of the mean water temperature to estimate cooking power. In our model cooking power is represented on the right hand side of the equation (3).

We also estimate standardized cooking power; which is defined as the cooking power when the temperature difference between mean fluid temperature and ambient temperature reaches 50 degrees; *i.e.* $\Delta T = T_f - T_{amb}$ [13].

4. Results and comparison with field testing

In figures (3.1) and (3.2) we show the numerical results (solid line) and experimental data (points) for the case of mean water temperature for two containers: an aluminum pressure pot and a stainless steel pot both equipped with aluminum sheets as reflectors. Figures (3.3) and (3.4) show the cooking power.

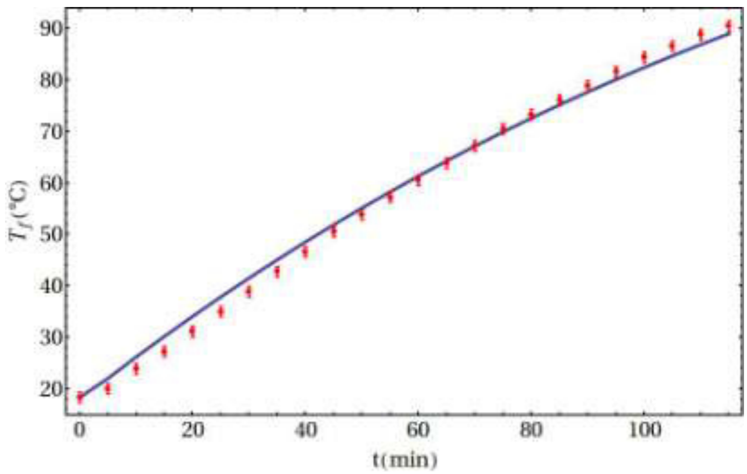

Figure 3.1: Mean water temperature for a water mass of 4.2 kg as a function of time; in this case we used an aluminum pressure pot. The numerical solution is represented by a solid line and the experimental data by dots with error bars. The relative error between the numerical and experimental data is 2.89%.

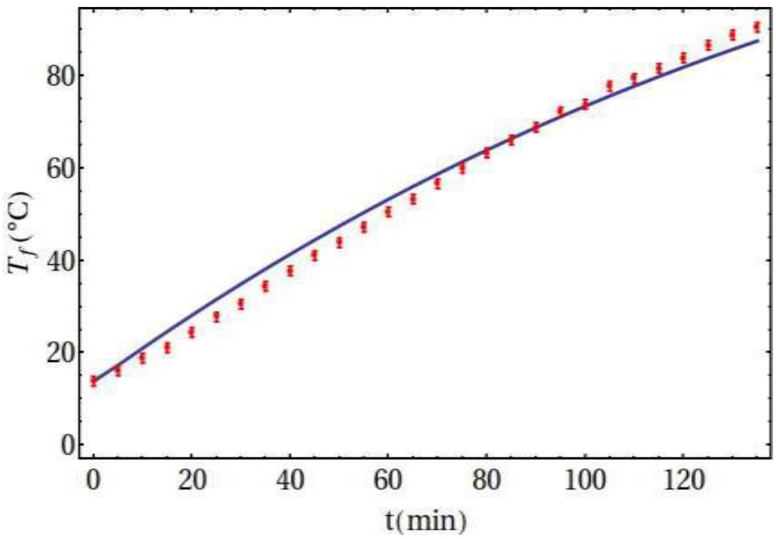

Figure 3.2: Mean water temperature for a water mass of 4.2 kg as a function of time; in this case we used a stainless steel pot. The numerical solution is represented by a solid line and experimental data by dots with error bars. The relative error between the numerical and experimental data is 4.43%.

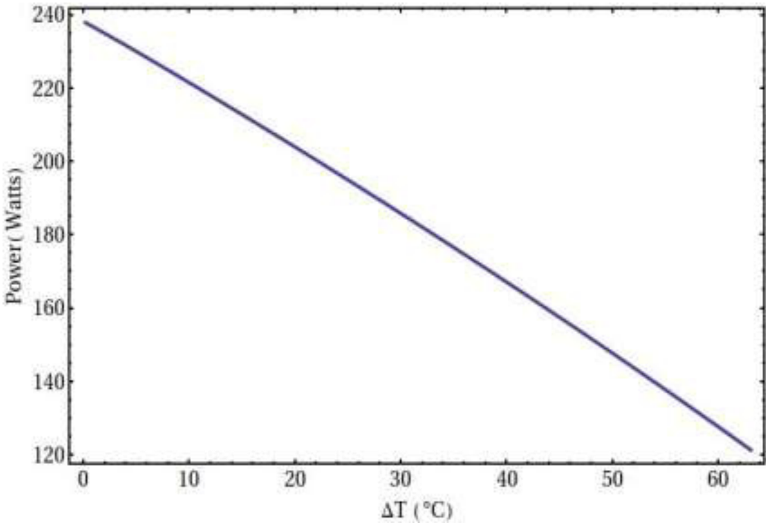

Figure 3.3: Cooking power as a function of temperature difference in the case of an aluminum pot. We can see that standardized cooking power is equal to 111.644 watts.

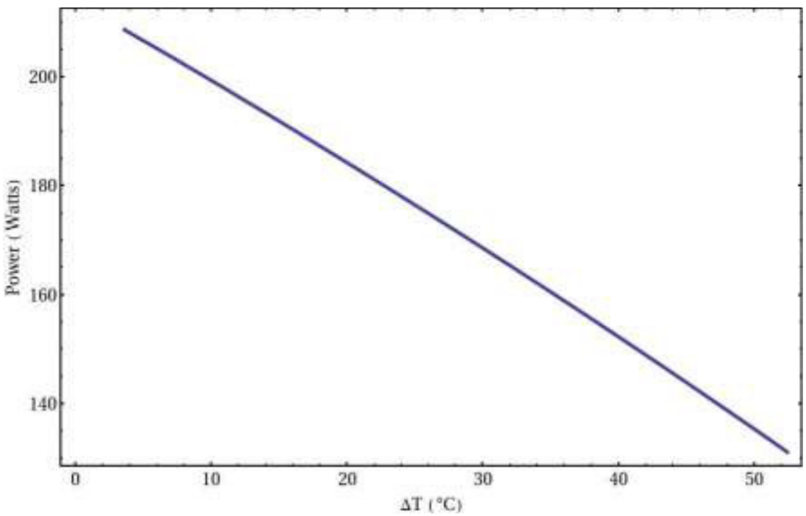

Figure 3.4: Cooking power as a function of temperature difference in the case of a stainless steel pot. We can see that the numerical standardized cooking power is equal to 102.364 watts.

4.1 Tables

Table 1: In this table we show the results for standardized cooking power and relative error between the numerical and experimental data for the case of mean water temperature.

Cases	Numerical standardized cooking power (watts)	Experimental standardized cooking power (watts)	Relative error between numerical and experimental data of mean water temperature (%)
SIM1	115.2	123.2	2.26
SIM2	104.3	118.6	4.43

5. Discussion

In this paper, we obtain a mathematical thermal model based on energy balance that is useful for simulating the thermal behavior of the solar cooker, “Jorhejpataranskua”. We calculated mean water temperature, mean absorber container temperature and mean surface reflectors temperature, numerically. However, we only compared the numerical solutions for mean water temperature with respect to experimental data, because we did not have reliable experimental data for the other two cases.

We found good agreement between the numerical and experimental solutions with a relative error below 4%. In addition, we used numerical results to calculate standardized cooking power for two different container pots: aluminum and stainless steel. In this case, we obtained satisfactory results with a relative error below 12% between the numerical and experimental data.

6. Conclusions

The mathematical thermal model developed in this paper generated good results in the case of mean water temperature, as this increased over time and nearly reached the boiling point. We also calculated standardized cooking power, which is important in determining the efficiency of our solar cooker.

This thermal model can be useful in calculating the standardized cooking power of solar cookers like the “Jorhejpataranskua”. It can also be useful for predicting some of the temperatures of interest in thermal analyses of solar cookers. Finally, it also allows us to calculate standardized cooking power without making field tests.

Finally, it is possible to vary such parameters as the collection area of the collector and absorber container, the optical properties of reflective sheets, and the selective film of the absorber, among others, as it demonstrated additional interesting thermal properties of a solar cooker.

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Appendix A. Numerical solution of coupled system of nonlinear differential equations in Mathematic 8

Example of the code used in Mathematica 8:

```
Sistemasim = FullSimplify[sistema = {Tr1'[t] == 1/(mr * cr) * (Q1rad - Q2rad - Q3conv - Q4rad - Q8conv - Q9rad - Q3conv2),
    Trf'[t] == 1/(mrf * crf) * (Q5rad + Q4rad - Q6conv - Q7rad + Q3conv2), Tf'[t] == 1/(mf * cf) * (Q8conv + Q9rad)}];
sol = NDSolve[{Sistemasim, Trf[0] == 33, Tr1[0] == 21, Tf[0] == 18.33}, {Trf, Tf, Tr1}, {t, 0, 150}, Method → "Automatic"]
```